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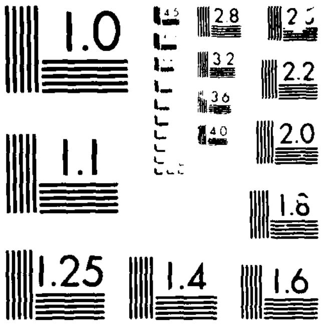

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REPORT NUMBER 86-06

AD-A168 810

FORCE EQUIVALENCE INDICES: ON ASSIGNING VALUES TO HETEROGENEOUS FORCES INVOLVED IN COMBINED ARMS COMBAT

US ARMY
TANK-
AUTOMOTIVE COMMAND

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June 1986

TACOM

SYSTEMS ANALYSIS DIVISION

Systems and Cost Analysis Directorate

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Techniques from graph theory are applied to the assigning of "value" to weapon systems involved in combined arms combat. The killer-victim matrix associated with imaginary battle is transformed into various types of coefficients reflecting lethality and survivability. Each of these matrices of coefficients is processed using a vector-convergence algorithm, outscores, and a combination of outscores and strong components; then checked against five acceptability criteria. Final selection of a method is based on variety of choice among these coefficents and mathematical simplicity.

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Table of Contents

Objectives

The goal of this study is to develop an algorithm which will assign normalized measures of potential to heterogeneous forces engaged in combined arms combat. The algorithm will meet the following criteria:

1. It produces reasonable results for any killer-victim matrix to which it is applied.

2. Its mathematical formulation is the simplest possible.

3. The interpretation of the above formulation does not run counter-intuitive to equations used in the physical sciences.

Introduction

This study was motivated by the need to revamp the computation of force ratios in the Force Comparison Model (FORCECOM). FORCECOM applied a series of multipliers to scores based on laboratory performance of the systems involved. The formulas used to derive these multipliers came from the historical analysis of Dupuy (4).

Research showed that analysts took two approaches to assigning scores to weapons. The first was to use the eigenvector corresponding to the largest eigenvalue of the adjusted killer-victim matrix. (See 2, ch. 30.) This approach, however, is limited to non-zero matrices and a certain class of non-negative matrices.

Other analysts, such as Johnsrud (5), developed non-linear methods. The ways in which they attained the desired non-linearity for their equations are not intuitively obvious. Under close examination, their algorithms violate accepted physical formulas.

The killer-victim matrix can serve as the set of coefficients for a system of linear difference equations. Noting this, one can see the applicability of the methods used with pulse processes, a branch of graph theory. Using ideas from this area of mathematics, the purpose of this paper is to suggest a more general linear algorithm, which satisfies the following criteria suggested by Johnsrud:

1. Arbitrary indexing of the weapons involved should not effect the results.

2. Having zero losses should not prevent solving the problems with acceptable results.

3. Killers of killers must have some weight.

4. Small changes in the scoreboard should produce small changes in the resulting values.

5. Adding either numbers or capabilities to a side should increase its computed strength relative to the opposing side.

Summary of Conclusions

The best available measure of a weapon system's "potential" is a geometric mean of its lethality value and its survivability value. These values are the outscores of the appropriately converted killer-victim matrix.

New models, which use either weapon system values or force ratios as drivers, should employ this methodology.

Graph Theory Concepts

A directed graph is a collection of points, or nodes, together with the arcs which connect the nodes. The graph is called directed because traffic along an arc is one-way. A path is a set of continuous arcs leading from one node to another. Two nodes are connected if there is a path from one to the other.

A set of nodes is said to be strongly connected if each member node has a path to every other node in the set. A strongly connected set is called a strong component if it contains all the nodes in a digraph which are strongly connected to the nodes in the set. Strong components partition the digraph's nodes into mutually exclusive sets. If these strong components are treated as nodes, a new digraph, called a condensation, can be derived. The condensation's arcs are found by connecting strong components with an arc if and only if a member node of one strong component is connected to a member node of the other strong component.

The killer-victim matrix for a battle has a directed graph associated with it. The nodes are the weapon system classes, and the weighted arcs are the numbers of the systems at the initial nodes "killed" per system at the terminal node. The strong components partition the collection of weapon system classes into groups of weapon systems which interact with each other and which should be treated as separate problems. Once the strong components have been identified, the killer-victim matrix should be converted into one of the coefficient matrices discussed under "Coefficients."

The following technique assumes that there is a fixed amount of "value" in any battle.

1. Partition the systems into strong components.

2. Construct the condensation of the systems.

3. Beginning with an estimate of equal value for each strong component, multiply the estimate vector by the adjusted condensation matrix until the change in the vector's values approaches zero. Use this last estimate of values.

4. Using the values obtained for the condensation, assign a percentage of the total value to each strong component.

5. Find the associated vectors for each strong component.

6. Compute the percentage of total value per weapon system by multiplying the weapon's percentage of strong component value by the strong component's percentage of total value.

If the battle has an irreducible matrix representation, it is itself the only strong component. The case then devolves to solving by the eigenvalue technique, using the coefficient matrix.

The following is an outline for the method of determining a vector of values for the condensation of a graph. While values for the strong components and, in some cases, the condensation (5, p. 77), may be found using eigenvalue techniques, this procedure will yield equivalent values if the eigenvalues of the matrix are less than one in magnitude.

1. Convert the killer-victim scoreboard into a condensed scoreboard.

   a. Add together the losses for all weapon systems within a strong component. Add together the initial numbers of weapon systems within a strong component.

   b. Convert into a coefficient matrix.

2. Let V(*) be the coefficient matrix and xV be the vector of current values. Obtain new vector values as follows:

   a. xV(k+1) = V(*) * xV(k).

   b. Initially, all values of the vector are set equal to one.

3. Repeat the computation in 2.a. until the maximum change is sufficiently close to zero.

4. Convert these final values to percentages by dividing each value in the vector by the sum of all the values.

5. Using the appropriate entries from the original killer-victim matrix, repeat the above steps for each of the strong components.

Another useful concept is that of the outscores, or the sum of entries in a row of a matrix. It is usually used to rank the participants of a tournament, the matrix involved having only 0's and 1's. If the percentage of lethality or survivability is substituted for the 1=win, and 0=loss of the tournament, one obtains the first iteration of the multiplication described above. The employment of the condensation might also be avoided since interaction plays a small part in computing outscores. The outscore, then, becomes attractive due to its simplicity.

In summary, the coefficients of the next chapter will be processed using each of the three algorithms:

1. Strong condensation and vector convergence.

2. Strong condensation and outscores.

3. Outscores only.

Coefficients

There are two main divisions in the types of coefficients used in this paper: lethality and survivability. Lethality scores are basically a percentage killed. They are either the percentage killed during the observed "combat", or the percentage expected to be killed at some calculated time. Survivability scores are the percentage surviving and are calculated by taking the transpose of the complement of the lethality score. The following is a list of descriptions.

1. Lethality Scores

a. Percentage of Contribution. The percentage of the victim's losses attributed to the killer.

b. Percent Killed. The percent of the victim killed by the killer.

c. Percent Killed by Minimum Annihilation Time. For each victim, total the percentages killed and compute the inverse. Multiply each of the kill percentages by the minimum of the inverses.

2. Survivability Scores

a. Percentage Surviving. For each percentage killed score, subtract from one and divide by the number of system types. Transpose the matrix.

b. Percentage Surviving at "End Time". Repeat above process using the percentage-killer-by-endtime scores.

c. Weighted Percentage Surviving. For each percentage-killed score, subtract from one and multiply by the percentage-of-contribution score. Transpose the matrix.

d. Weighted Percentage Surviving at "Endtime". Repeat the above process using the percentage-killed-by-end-time scores in place of percentage-killed scores.

e. Percentage Surviving vs. Opponents. For each percentage-killed score, subtract from one if killer and victim have different colors and divide by the number of opposing system types. Transpose the matrix.

f. Percentage Surviving vs. Opponents at "Endtime". Repeat the above process using the percentage-killed-by-endtime scores.

Each of the above was computed on a "per weapon" basis. All entries in the killer-victim scoreboard were converted to a percentage-of-opponent-killed per killer system as part of the forming of the coefficients.

Methodology

The coefficients in the previous chapter were tested with the data found in Table 1. The following is a list of the tests used to check compliance with Johnsrud's criteria.

1. Arbitrary indexing is permitted.

a. In the case of vector convergence, a dummy vector is used during matrix multiplication, and this allows for interchange of rows and columns.

b. The formulation of condensations and the computation of outscores is unaffected by the interchange of rows and columns.

2. Reducible matrices give reasonable results.

a. In the case of vector convergence, the sequence of vectors does not diverge.

b. Where condensation is used, strong components which are intuitively more effective will have larger percentages of battle value.

c. Intuitively more effective systems will have larger outscores than other systems.

3. Killers of killers should have non-zero values. This is met by all three algorithms.

4. Small scoreboard changes should produce small changes in results.

a. For each side, infantry is allowed to kill one artillery piece.

b. Changes in the force ratio will be less than five percent.

5. Adding something to a side increases its worth relative to the other side.

a. The initial number of Blue armor was increased ten percent.

b. The force ratio (FR = R/B) will decrease.

Results

The results of the tests for the coefficients, using vector convergence, condensation and outscores, and simple outscores, are in Tables 2, 3, and 4, respectively. Of those coefficients passing all criteria, three used vector convergence, two used condensation with outscores, and eight used outscores only.

As a sidelight, two versions of the Potential-Antipotential Algorithm were checked as well. The original version failed to lower the force ratio when Blue's armor was increased. The second version, which converted the matrix entries to percentage killed per system instead of per category, passed all criteria.

Analysis

Of the three methods for determining the vectors of scores, none is really a clear candidate. Vector convergence is an extension of both the eigenvalue method and of accepted process theory techniques. It does the most to show the dependence of a system's value on the value of its opponents. It is also the most complex algorithm to implement, and has the fewest coefficients from which to choose.

The manner is which the outscore has been employed is a logical extension of its use in graph theory. The technique can be used on any matrix involving only killers. It does, however, eliminate the idea of worth of a target. Outscores offer the largest selection of possible coefficients, and have the simplest implementation.

The combination of outscores and condensation is the least desirable choice. While trying to offer the best of the other two alternatives, it still eliminates target worth. In addition to taking almost the same amount of code to implement, there are fewer available coefficients from which to choose than in the case of plain outscores.

The scores obtained above were multiplied by their corresponding number of weapon systems to yield the "potential" of each weapon category. For each type of score, the categories were ranked: 1=highest potential; 10=lowest. The results of .e outscore methods were correlated with the result of the vector-convergence method using Spearman's formula. Both percentage-surviving and percentage-surviving-at-endtime showed a high correlation (0.903) of the results of the outscores alone and the vector convergence. For these two coefficients, the outscore method is preferred due to its simplicity.

As for choosing a coefficient, success points to the percentage-surviving and the percentage-surviving-at-endtime scores. The fact that these are both survival coefficients opens the question of which comes first, survivability or lethality. This could be avoided by making the score a function of both lethality and survivability. Since the scores have been normalized, a weighted geometric is the best solution. (Normalization prevents the use of an arithmetic mean.) The obvious pairings are percentage-killed and percentage-surviving, and percentage-killed-by-endtime and percentage-surviving-at-endtime. These geometric means also satisfy the criteria for coefficients.

For the above four coefficients, (using outscores), it is easy to prove that Criterion #4 will hold if all systems are killed by some opposing system. Suppose that the number of weapon system x has been increased by a factor of A>1. Then, consider two cases. If using a lethality score, the total of the side having system x will remain unchanged, as the factor A in the number of type x systems will cancel with the factor A in the denominator of x's score (s(x)). The total for the opposing side will decrease as the coefficients of all killers versus weapon x will be divided by A. (If no type x systems were killed, the ratio remains unchanged.)

In the case of a survivability score, the opposing total will behave as in lethality scores. The side with system x, however, will increase by (A-1) * s(x). In either type of score, the Force Ratio will change in favor of the side owning system x, (as long as it is killed by something.)

If an entry in the killer-victim scoreboard is increased, the lethality score of the killer will obviously be increased. This raises the total of the side with the killer, while leaving the victim's side constant. In the case of survivability, this lowers the score of the victim and his side, while the killer's side constant. In either case, the change in the Force Ratio favors the killer. Likewise, a decrease in a killer-victim matrix entry favors the victim's side.

To prove that Criterion #5 will hold, consider the following equation:

$$FR(delta) = FR(old) - FR(new)$$

$$\frac{(Rsum(old)*Bsum(new)) - (Rsum(new)*Bsum(old))}{Bsum(old) * Bsum(new)}$$

It is easy to show that FR(delta) is bounded by two expressions involving changes in the killer-victim matrix, and that these expressions both go to zero as the changes in the killer-victim matrix go to zero. Therefore, if changes in the killer-victim matrix are small, the change in the Force Ratio must also be small.

Conclusions

First, the present methods of computing weapon system values have serious drawbacks, both from a mathematical view and a practical view, and should be replaced.

Second, the concept of outscores, using an appropriate version of the killer-victim matrix, provides the simplest mathematical tool for assigning weapon system values.

Third, a weighted geometric mean of lethality and survivability scores will provide the best measure of a weapon system's worth.

Recommendations

The outscore process should be applied to the results of simulations to demonstrate its validity.

The appropriate constants for the weighted geometric mean of lethality and survivability scores should be found.

Until the above validation takes place, the Potential-Antipotential method should be replaced with the adjusted version tested in this paper.

Table 1. Augmented Killer-Victim Scoreboard

		Victims									
		Blue					Red				
Killers		Inf	Arm	ADA	AH	FA	Inf	Arm	ADA	AH	FA
Red	Inf	110	45	10	10	0	0	0	0	0	0
	Arm	100	85	5	0	0	0	0	0	0	0
	ADA	0	0	0	10	0	0	0	0	0	0
	AH	30	15	15	5	0	0	0	0	0	0
	FA	70	15	20	0	10	0	0	0	0	0
Blue	Inf	0	0	0	0	0	100	30	10	0	0
	Arm	0	0	0	0	0	100	80	20	0	0
	ADA	0	0	0	0	0	0	0	0	5	0
	AH	0	0	0	0	0	50	20	15	3	0
	FA	0	0	0	0	0	80	10	20	0	5
Initial Systems		1200	300	200	50	50	1000	500	100	10	60

Table 2. FEIs and Force Ratios Using Vector Convergence

Weapon Scores

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
CNTRBN											
1	1	7.46	124.	329.	2230.	21.2	4.29	175.	1400.	1860.	1.07
2	1	6.77	125.	329.	2250.	21.4	4.25	176.	1410.	1870.	1.06
3	1.	1.22	19.8	52.4	106.	3.95	.713	27.8	223.	113.	1.2

Failed criterion #4 since the FR change in #3 was 12%.

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
%KILLED											
1	1.	7.66	164.	426.	143.	16.9	2.06	152.	994.	285.	.93

Failed criterion #2 since artillery was rated less than other systems.
Other criteria not tested.

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
RxTmin											
1	1.	7.66	164.	426.	143.	16.9	2.06	152.	994.	285.	.93

Failed criterion #2 since artillery was rated less than other systems.
Other criteria not tested.

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
%SURVNG											
1	1.	.979	.965	.932	4.51	1.01	1.01	.993	.96	4.76	.975
2	1	.982	.965	.932	4.51	1.01	1.01	.994	.96	4.75	.96
3	1.	.984	.968	.952	1.	1.	1.01	.977	.974	1.01	.937

Passed all criteria.

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
%SURVTm											
1	1.	1.	.999	.999	4.	1.	1.	.999	.999	4.	.949
2	1.	1.	.999	.999	4.	1.	1.	.999	.999	4.	.934
3	1.	1.	.999	.999	1.	1.	1.	.999	.999	1.	.928

Passed all criteria.

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
W%SURV											
1	1.	.951	2.83	2.03	5.26	.307	.321	.393	.665	7.24	.392
2	1.	.957	2.83	2.03	5.24	.304	.315	.389	.664	7.2	.384
3	1.	.799	2.48	1.51	.938	.598	.367	.696	.542	1.23	.452

Failed criterion #4 since the FR change in #3 was 15%.

Table 2. FEIs and Force Ratios Using Vector Convergence (Continued)

Weapon Scores

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
W%SRVTm											
1	1.	1.22	.737	1.13	4.14	.99	1.26	.926	1.03	4.15	.997
2	1.	1.22	.737	1.13	4.14	.99	1.26	.926	1.03	4.15	.979
3	1.	1.	.995	.996	.995	1.	1.	.998	.993	.997	.928
Failed criterion #4 since the FR change in #3 was 7%.											
%SRVOPPS											
1	1.	.973	.958	.913	2.37	.251	.251	.23	.187	3.26	.323
2	1.	.977	.958	.913	2.17	.249	.249	.228	.185	2.98	.309
3	1.	.978	.956	.935	1.	.365	.375	.327	.323	.375	.343
Failed criterion #4 since the FR change in #3 was 6%											
%SRVOPTm											
1	1.	.965	.948	.89	3.2	.249	.249	.223	.169	4.59	.357
2	1.	.971	.948	.89	3.19	.247	.247	.221	.166	4.57	.349
3	1.	.972	.945	.92	.999	.36	.373	.313	.309	.373	.340
Passed all criteria.											

Table 3. FEIs and Force Ratios Using Condensation with Outscores

	Weapon Scores										
	Blue					Red					
Case No.	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Force Ratio
%CNTRBN											
1	1	7.05	5.59	32.2	144.	2.21	3.41	7.15	123.	120.	.986
2	1	6.4	5.59	32.2	145.	2.21	3.41	7.15	123.	121.	.987
3	1	5.64	4.48	25.8	41.7	1.9	2.73	5.73	98.9	38.9	1.0
Failed criterion #5 since the FR change in #2 favored Red.											
%KILLED											
1	1	7.08	11.5	49.8	17.	2.27	3.62	9.23	108.	29.5	.913
2	1	6.43	11.5	49.8	17.	2.21	3.38	9.23	113.	34.	.892
3	1	6.65	10.8	46.8	33.3	2.22	3.4	8.67	108.	29.5	.816
Failed criterion #4 since the FR change in #3 was 11%.											
RxTmin											
1	1	7.08	11.5	49.8	17.	2.27	3.62	9.23	115.	29.5	.913
2	1	6.43	11.5	49.8	16.7	2.21	3.38	9.23	113.	33.4	.889
3	1	6.65	10.8	46.8	33.3	2.22	3.4	8.67	108.	29.5	.816
Failed criterion #4 since the FR change in #3 was 11%.											
%SURVNG											
1	1	.979	.965	.932	4.57	1.01	1.01	.993	.959	4.79	.974
2	1	.982	.965	.932	4.16	1.01	1.01	.993	.959	4.37	.957
3	1	.984	.968	.952	1.	1.	1.01	.977	.973	1.01	.937
Passed all criteria.											
%SURVTm											
1	1	1.	.999	.999	4.	1.	1.	1.	.999	4.	.949
2	1	1.	.999	.999	4.	1.	1.	1.	.999	4.	.934
3	1	1.	.999	.999	1.	1.	1.	.999	.999	1.	.928
Passed all criteria.											
W%SURV											
1	1	.957	2.72	2.08	4.57	.388	.436	.47	.658	6.29	.439
2	1	.964	2.72	2.08	4.54	.38	.421	.462	.658	6.24	.425
3	1	.82	2.46	1.55	.964	.61	.426	.703	.49	1.11	.468
Failed criterion #4 since the FR change in #3 was 7%.											

Table 3. FEIs and Force Ratios Using Condensation with Outscores (Continued)

	Weapon Scores										
	Blue					Red					
Case No.	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Force Ratio
W%SRVTm											
1	1	1.17	.772	1.29	4.29	.979	1.2	.893	1.29	4.3	.977
2	1	1.17	.772	1.29	4.29	.979	1.2	.893	1.29	4.3	.96
3	1	1.	.997	.997	.997	1.	1.	.998	.996	.999	.928
Failed criterion #4 since the FR change in #3 exceeded 5%.											
%SRVOPPS											
1	1	.973	.956	.914	2.39	.252	.252	.231	.187	3.29	.325
2	1	.978	.956	.914	2.18	.249	.25	.228	.185	2.99	.309
3	1	.978	.956	.933	1.	.366	.376	.329	.323	.377	.344
Failed criterion #4 since the FR change in #3 was 6%.											
%SRVOPTm											
1	1	.966	.945	.892	3.87	.25	.25	.223	.169	5.55	.382
2	1	.972	.945	.892	3.85	.248	.248	.221	.166	5.52	.373
3	1	.972	.945	.915	1.	.363	.375	.316	.308	.377	.342
Failed criterion #4 since the FR change in #3 was 10%											

Table 4. FEIs and Force Ratios Using Outscores Only

	Weapon Scores										
	Blue					Red					
Case No.	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Force Ratio
%CNTRBN											
1	1	7.05	5.59	32.2	58.	2.21	3.41	7.15	123.	51.2	.999
2	1	6.4	5.59	32.2	58.	2.21	3.41	7.15	123.	51.2	.999
3	1	5.64	4.48	25.8	41.7	1.9	2.73	5.73	98.9	38.9	1.0
Failed criterion #5 since the FR change in #2 favored Red.											
%KILLED											
1	1	7.08	11.5	49.8	35.4	2.27	3.62	9.23	115.	31.4	.813
2	1	6.43	11.5	49.8	35.4	2.21	3.38	9.23	113.	31.1	.791
3	1	6.65	10.8	46.8	33.3	22.2	3.4	8.67	108.	29.5	.816
Passed all criteria.											
RxTmin											
1	1	7.08	11.5	49.8	35.4	2.27	3.62	9.23	115.	31.4	.813
2	1	6.43	11.5	49.8	35.4	2.21	3.38	9.23	113.	31.1	.791
3	1	6.65	10.8	46.833.3		22.2	3.4	8.67	108.	29.5	.816
Passed all criteria.											
%SURVNG											
1	1	.984	.968	.952	1.	1.	1.01	.977	.973	1.01	.937
2	1	.987	.968	.952	1.	1.	1.01	.977	.973	1.01	.921
3	1	.984	.968	.952	1.	1.	1.01	.977	.973	1.01	.937
Passed all criteria.											
%SURVTm											
1	1	.999	.999	.998	1.	1.	1.	.999	.999	1.	.928
2	1	.999	.999	.998	1.	1.	1.	.999	.999	1.	.913
3	1	.999	.999	.998	1.	1.	1.	.999	.999	1.	.928
Passed all criteria.											
W%SURV											
1	1	.819	2.46	1.55	.957	.61	.426	.704	.49	1.32	.474
2	1	.826	2.46	1.55	.958	.605	.415	.699	.49	1.32	.462
3	1	.82	2.46	1.55	.964	.61	.426	.703	.49	1.11	.468
Passed all criteria.											

Table 4. FEIs and Force Ratios Using Outscores Only (Continued)

Weapon Scores

Case No.	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
W%SRVTm											
1	1.	1.	.997	.997	.997	1.	1.	.998	.996	.999	.928
2	1.	1.	.997	.997	.997	1.	1.	.998	.996	.999	.913
3	1.	1.	.997	.997	.997	1.	1.	.998	.996	.999	.928
Passed all criteria.											
%SRVOPPS											
1	1.	.978	.956	.933	1.	.366	.376	.329	.323	.377	.344
2	1.	.982	.956	.933	1.	.364	.374	.327	.321	.373	.336
3	1.	.978	.956	.933	1.	.366	.376	.329	.323	.377	.344
Passed all criteria.											
%SRVOPTm											
1	1.	.972	.945	.915	1.	.363	.375	.316	.308	.377	.342
2	1	.978	.945	.915	1.	.361	.374	.314	.306	.374	.335
3	1.	.972	.945	.915	1.	.363	.375	.316	.308	.377	.342
Passed all criteria.											

Table 5. FEIs and Force Ratios Using Different Methods

	Weapon Scores										
	Blue					Red					
Method	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Force Ratio
%CNTRBN											
Vec Conv	1	7.46	124.	329.	2230.	21.2	4.29	175.	1400.	1860.	1.07
Cond/OSs	1	7.05	5.59	32.2	144.	2.21	3.41	7.15	123.	121.	.986
Outscore	1.	7.05	5.59	32.2	58.	2.21	3.41	7.15	123.	51.2	.999
%KILLED											
Vec Conv	1	7.66	164.	426.	143.	16.9	2.06	152.	994.	285.	.93
Cond/OSs	1	7.08	11.5	49.8	17.	2.27	3.62	9.23	115.	33.9	.913
Outscore	1	7.08	11.5	48.8	35.4	2.27	3.62	9.23	115.	31.4	.813
RxTmin											
Vec Conv	1	7.66	164.	426.	143.	16.9	2.06	152.	994.	285.	.929
Cond/OSs	1	7.08	11.5	49.8	17.	2.27	3.62	9.23	115.	33.9	.913
Outscore	1	7.08	11.5	49.8	35.4	2.27	3.62	9.23	115.	31.4	.813
%SURVNG											
Vec Conv	1	.979	.965	.932	4.51	1.	1.01	.993	.96	4.76	.975
Cond/OSs	1	.979	.965	.932	4.57	1.01	1.01	.993	.959	4.79	.974
Outscore	1	.984	.968	.952	1.	1.	1.01	.977	.973	1.01	.937
%SURVTm											
Vec Conv	1	1.	.999	.999	4.	1.	1.	1.	.999	4.	.949
Cond/OSs	1	1.	.999	.999	4.	1.	1.	1.	.999	4.	.949
Outscore	1	.999	.999	.998	1.	1.	1.	.999	.999	1.	.928
W%SURV											
Vec Conv	1	.951	2.83	2.03	5.26	.307	.321	.393	.665	7.24	.392
Cond/OSs	1	.957	2.72	2.08	4.57	.388	.436	.47	.658	6.29	.439
Outscore	1	.819	2.46	1.55	.957	.61	.426	.704	.49	1.32	.474
W%SRVTm											
Vec Conv	1	1.22	.737	1.13	4.14	.99	1.26	.926	1.03	4.15	.997
Cond/OSs	1	1.17	.772	1.29	4.29	.979	1.2	.893	1.29	4.3	.977
Outscore	1	1.	.997	.997	.997	1.	1.	.998	.996	.999	.928

Table 5. FEIs and Force Ratios Using Different Methods (Continued)

Weapon Scores

Method	Blue INF	Blue ARM	Blue ADA	Blue AH	Blue FA	Red INF	Red ARM	Red ADA	Red AH	Red FA	Force Ratio
%SRVOPPS											
Vec Conv	1	.973	.958	.913	2.37	.251	.251	.23	.187	3.26	.323
Cond/OSs	1	.973	.956	.914	2.39	.252	.252	.231	.187	3.29	.325
Outscore	1	.978	.956	.933	1.	.366	.376	.329	323	.377	.344
%SRVOPTm											
Vec Conv	1	.965	.948	.89	3.2	.249	.249	.223	.169	4.59	.357
Cond/OSs	1	.966	.945	.892	3.87	.25	.25	.223	.169	5.55	.382
Outscore	1	.972	.945	.915	1.	.363	.375	.316	.308	.377	.342

Table 6. Ranks and Spearman Scores Using Different Methods

	Weapon Category Ranks										
	Blue					Red					
Method	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Spearman Score
%CNTRBN											
Vec Conv	10	8	3	6	2	4	9	5	7	1	
Cond/OSs	8	4	9	6	2	3	5	10	7	1	.406
Outscore	8	4	9	6	2	3	5	10	7	1	.406
%KILLED											
Vec Conv	9	8	1	2	7	4	10	5	6	3	
Cond/OSs	7	4	2	1	10	3	6	9	8	5	.564
Outscore	8	4	2	1	7	3	6	10	9	5	.552
RxTmin											
Vec Conv	9	8	1	2	7	4	10	5	6	3	
Cond/OSs	7	4	2	1	10	3	6	9	8	5	.564
Outscore	8	4	2	1	7	3	6	10	9	5	.552
%SURVNG											
Vec Conv	1	4	7	9	6	2	3	8	10	5	
Cond/OSs	1	4	7	9	6	2	3	8	10	5	1.00
Outscore	1	4	5	9	8	2	3	6	10	7	.903
%SURVTm											
I	1	4	7	9	6	2	3	8	10	5	
Cond/OSs	1	4	7	9	6	2	3	8	10	5	1.00
Outscore	1	4	5	9	8	2	3	6	10	7	.903
W%SURV											
Vec Conv	1	5	2	8	6	4	7	9	10	3	
Cond/OSs	1	5	2	8	6	3	7	9	10	4	.988
Outscore	1	4	3	7	9	2	5	8	10	6	.818
W%SRVTm											
Vec Conv	1	4	7	9	6	2	3	8	10	5	
Cond/OSs	1	4	7	9	6	2	3	8	10	5	1.00
Outscore	1	4	5	8.5	8.5	2	3	6	10	7	.888
%SRVOPPS											
Vec Conv	1	2	5	8	7	3	6	9	10	4	
Cond/OSs	1	2	5	8	7	3	6	9	10	4	1.00
Outscore	1	3	4	7	6	2	5	8	10	9	.806

Table 6. Ranks and Spearman Scores Using Different Methods (Continued)

	Weapon Category Ranks										
	Blue					Red					
Method	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Spearman Score
%SRVOPTm											
Vec Conv	1	2	5	8	6	4	7	9	10	3	
Cond/OSs	1	3	6	8	5	4	7	9	10	2	.976
Outscore	1	3	4	7	6	2	5	8	10	9	.709

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